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Runge-Kutta based calculation of turbulent forced convection flows of metallic liquids through tubes

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Abstract

Purpose – To provide a suitable linkage of a computational fluid dynamics code and a shape optimization code for the augmentation of local heat transfer coefficients in forced convection channels normally used in the cooling of electronic equipment.

Design/methodology/approach – A parallel-plate channel with a discrete array of heat sources embedded in one wall, while the other wall is insulated, constitutes the starting model. Using water as coolant, the objective is to optimize the shape of the channel employing a computerized design loop. The two-part optimization problem is constrained to allow only the unheated wall to deform, while keeping the same inlet shape and observing a maximum pressure drop constraint.

Findings – First, the results for the linearly deformed unheated wall show significant decrease compared with the wall temperatures of the heated wall, with the maximum wall temperature occurring slightly upstream of the outlet. Second, when the unheated wall is allowed to deform nonlinearly, a parabolic-like shaped wall is achieved where the maximum wall temperature is further reduced, with a corresponding intensification in the local heat transfer coefficient. The effectiveness of the computerized design loop is demonstrated in complete detail.

Originality/value – This paper offers a simple technique for optimizing the shapes of forced convection channels subjected to pre-set design constraints.

Keywords Optimization techniques, Convection, Cooling systems, Electronic equipment and components

Paper type Research paper

Nomenclature

c_p = specific heat at constant pressure
D = tube diameter
f = friction factor
h = local convection coefficient
k = thermal conductivity
l^+ = dimensionless mixing length (equation (3b))
$\dot{m}$ = mass flow rate
Nu = local Nusselt number, hD/k
Pr = Prandtl number, ν/α
Pr_t = turbulent Prandtl number, $\varepsilon_M/\varepsilon_H$
Q_t = total heat transfer rate (equation (13))
r = radial coordinate
R = tube radius

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R^+ = dimensionless value of r (equation (2))
Re = Reynolds number, $u_m D/\nu$
T = time-average temperature
u = time-average velocity
u_m = mean velocity
u_τ = shear velocity, $(\tau_w/\rho)^{1/2}$
u^+ = dimensionless value of u (equation (2))
x = axial coordinate
y = radial distance measured from the tube wall
y^+ = dimensionless value of y (equation (2))
z = dimensionless value of z (equation (8c))

Greek letters
α = thermal diffusivity
ε_H = eddy diffusivity of heat
ε_M = eddy diffusivity of momentum
η = dimensionless value of r (equations (8a)-(8e))
$\bar{\eta}$ = transformed η (equations (17a) and (17b))
θ = dimensionless value of T (equations (7a)-(7c))
μ = dynamic viscosity
ν = kinematic viscosity
ρ = density
τ = shear stress

Subscripts
b = mean bulk
e = entrance
i = line
w = tube wall
∞ = fully developed

Introduction

The transport of high temperature metallic liquids inside tubes and channels is of paramount interest in metallurgical and manufacturing engineering. In particular, metal liquid transport is a key component of the modern foundry industry and has been used for a number of years in the nuclear industry (Jones, 1986). Most of the applications in the non-ferrous industry are related to the transport of liquid alloys through refractory lined steel pipes from furnace to furnace or to feed intermediate vessels or casting stations (Neff and Cooper, 1990). Both, low pressure die-casting and permanent mould casting machines are fed in this manner (Sajdak *et al.*, 1985). While rectangular-shaped channels are common for transporting non-ferrous metals and their alloys, annular ducts are normally used for liquid sodium and lithium (Jones, 1986). Small amounts, in the order of 1,000 lbs, of aluminum may be transferred through refractory lined steel pipes with a loss of 1°C/m (Neff, 1986). This low level of heat loss impacts directly on productivity and quality, since less superheat is required in the furnace which results into shorter processing times and less oxidation and dross formation (due to the lower casting temperature associated with less superheat).

To determine dependable heat transfer rates for this important class of internal convective flow, a number of studies have been conducted based on numerical simulations and experimental observations in the laboratory. A brief account of the relevant investigations on this topic, considering fully developed velocity profiles and constant properties, will be given here. Notter and Sleicher (1972) presented an analytical solution for the turbulent Graetz problem following the ideas of the counterpart solution performed in the pioneering works by Graetz for laminar flows. These authors solved the Sturm-Liouville problem numerically for the range $10^4 < Re < 10^6$ and $0 < Pr < 10^4$. Chen and Chiou (1981) examined the behavior of turbulent heat transfer to metallic liquids for various combinations of Re and Pr in a tube using finite-difference techniques. The governing equations were solved by Patankar's methodology using a forward marching procedure with 20 and 30 nodes deployed in the cross-stream direction. Results were reported for Re up to 5×10^5 and $Pr \ll 0.1$. Lee (1982) investigated the important effect of axial fluid conduction for

a situation of a metallic liquid flowing turbulently in a tube with uniform wall temperature. The calculations led to the conclusion that this effect is significant in the thermal entry region only for values of the Peclet number lower than 100, but it is negligible in the thermally developed region.

The main focus of the present investigation is to analyze numerically the turbulent forced convection of fluids covering the low spectrum of the Prandtl number ($Pr \ll 0.1$) in the thermal entry region of a circular tube considering constant properties. Particular attention will be given to the case of uniform wall temperature and fully developed, turbulent velocity profile.

An appraisal of the conservation equations reveals that the choice of the models for turbulent momentum and energy transfer is of considerable importance and perhaps is the crucial aspect of the analysis in order to match quantitatively the numerical predictions with the experimental data. In view of this, Cebecci's (1973) version of von Karman's mixing length turbulence model has been adopted in this study. Furthermore, in discussing the form of the turbulent Prandtl number for low Prandtl number fluids ($Pr \ll 0.1$), the models recommended by Azer and Chao (1960) and Dwyer and Tu (1969) are employed.

The present study seeks to adapt the finite volume method of lines (FVMOL) to the problem of turbulent forced convection of metallic fluids flowing inside a circular tube. This particular problem is characterized by a one-dimensional ordinary differential momentum equation and a two-dimensional partial differential energy equation. Accordingly, using the Runge-Kutta integration algorithm for the momentum equation and the associated system of energy equations of first order gives directly the velocity profile and the mean bulk temperature distribution in the metallic liquid. Comparison of the theoretical distributions of mean bulk temperatures and local Nusselt numbers with existing numerical and experimental data are made in both the entrance and the fully developed thermal regions of the tube. In general, concordance is quite satisfactory bearing in mind that in this investigation coarse grids with only ten unequal intervals in the cross-stream direction were used.

Problem statement

Consider a metallic liquid ($Pr \ll 0.1$), at an elevated uniform temperature, entering the heat exchange region of a thin-walled tube. The flow will be assumed to be turbulent and fully developed and the internal surface of the tube is considered to be isothermal.

Integration of the dimensionless equation for conservation of momentum:

$$\left(1 + \frac{\varepsilon_M}{\nu}\right)\frac{du^+}{dy^+} = 1 - \frac{y^+}{R^+} \tag{1}$$

subjected to the boundary condition $u^+(0) = 0$, yields the turbulent velocity profile in the metallic liquid. The participating dimensionless variables are defined as:

$$u^+ = \frac{u}{u_\tau}, \quad y^+ = \frac{yu_\tau}{\nu}, \quad R^+ = \frac{Ru_\tau}{\nu} \tag{2}$$

The eddy diffusivity of momentum, ε_M, is given by Von Karman's mixing length theory incorporating Cebecci's (1973) modification:

$$\frac{\varepsilon_{\mathrm{M}}}{\nu} = (l^{+})^{2}\left|\frac{\mathrm{d}u^{+}}{\mathrm{d}y^{+}}\right| \tag{3a}$$

where l^{+} defines the mixing length and is given by:

$$l^{+} = R^{+}[0.14 - 0.08\eta^{2} - 0.06\eta^{4}]\left[1 - \exp\left(-\frac{y^{+}}{26}\right)\right] \tag{3b}$$

Once the turbulent velocity profile has been computed, the pressure drop Δp over a fixed tube length, L, may be determined from the friction factor:

$$f = \frac{8}{(u_{\mathrm{m}}^{+})^{2}} \tag{4}$$

where u_{m}^{+}, the dimensionless mean velocity, is computed from:

$$u_{\mathrm{m}}^{+} = 2\int_{0}^{1} u^{+}\eta\,\mathrm{d}\eta \tag{5}$$

The dimensionless energy conservation equation assuming fully developed turbulent hydrodynamics inside a smooth circular tube is given, in dimensionless form, by:

$$Re\frac{u^{+}}{u_{\mathrm{m}}^{+}}\frac{\partial\theta}{\partial z} = \frac{4}{\eta}\frac{\partial}{\partial\eta}\left[\eta\left(\frac{1}{Pr} + \frac{\varepsilon_{\mathrm{M}}/\nu}{Pr_{\mathrm{t}}}\right)\frac{\partial\theta}{\partial\eta}\right] \tag{6}$$

The main assumptions considered to write equation (6) are:

- the thermophysical properties of the metallic liquid are constant; and
- axial fluid conduction is negligible.

The boundary conditions are given by:

$$\theta(0,\eta) = 1 \tag{7a}$$

$$\frac{\partial\theta}{\partial\eta}(z,0) = 0 \tag{7b}$$

$$\theta(z,1) = 0 \tag{7c}$$

The dimensionless variables and parameters in equations (6)-(7c) are expressed by:

$$\theta = \frac{T - T_{\mathrm{w}}}{T_{\mathrm{e}} - T_{\mathrm{w}}} \tag{8a}$$

$$\eta = \frac{r}{R} \tag{8b}$$

$$z = \frac{x}{D} \tag{8c}$$

$$Re = \frac{u_m D}{\nu} \tag{8d}$$

$$Pr = \frac{\nu}{\alpha} \tag{8e}$$

As far as the eddy diffusivity of momentum is concerned, the expression for the ε_M distribution is given in equations (3a) and (3b).

According to Reynolds (1975), the existing models for the turbulent Prandtl number, $Pr_t = \varepsilon_M/\varepsilon_H$, range from purely empirical considerations to formal mathematical analysis based on the Reynolds stress equation. Among the candidate models available, the present calculations will rely on the popular models proposed by:

- Azer and Chao (1960):

$$Pr_t = \frac{1 + \dfrac{380 f(y/R)}{(Re\,Pr)^{0.58}}}{1 + \dfrac{135 f(y/R)}{(Re)^{0.45}}} \tag{9a}$$

where:

$$f(y/R) = \exp\left[-\left(\frac{y}{R}\right)^{0.25}\right] \tag{9b}$$

and

- Dwyer and Tu (1969):

$$\frac{1}{Pr_t} = 1 - \frac{1.82}{Pr(\varepsilon_M/\nu)_{max}^{1.4}} \tag{10a}$$

where:

$$(\varepsilon_M/\nu)_{max} = 4 + 0.002897 Re^{0.919} \tag{10b}$$

respectively.

Once the dimensionless mean bulk temperature:

$$\theta_b(z) = \frac{\int_0^1 u^+ \theta \eta \mathrm{d}\eta}{\int_0^1 u^+ \eta \mathrm{d}\eta} \tag{11}$$

has been accurately computed, the total heat flow, Q_t, transferred in a certain length of the heat exchange region, L, may be calculated in two different ways:

(1) Indirectly by means of the local convection coefficient h, usually expressed in dimensionless form by the local Nusselt number. The latter may be computed using:

$$Nu(z) = \frac{-2\dfrac{\partial \theta}{\partial \eta}(z,1)}{\theta_b(z)} \tag{12}$$

Or

(2) Directly, by simply computing the change in enthalpy applying a global energy balance:

$$Q_t = \dot{m} C_p [T_e - T_b(L)] \tag{13}$$

where the magnitude of T_b is obtained from equation (11).

In light of the foregoing, the advantage offered by the second procedure is self-explanatory, because knowledge of the mean bulk temperature T_b at a certain axial position $z = L/D$ leads to the total heat transferred in that specific region $0 < z < L/D$.

Hybrid method of solution

The computation of the turbulent velocity profile in equation (1) is rather straight-forward and it may be accomplished utilizing a standard fourth-order Runge-Kutta algorithm.

Next, the numerically determined velocity is introduced in equation (6) in order to calculate the axial development of the temperature field, $T(x, r)$, in the tube. In general, a partial differential equation of parabollic type, like equation (4), has to be solved numerically. Using any discrete solution method (Morton and Mayers, 1994), this equation can be approximated at the point i, j in the computational domain by an algebraic equation accounting for a five-point molecule. Thus, having such an algebraic equation for each interior discrete point within the fluid domain, the main task is to solve a system of algebraic equations for the unknown quantities θ_{ij} by direct solution methods having explicit or implicit characteristics.

Alternatively, we propose a hybrid procedure based on the combination of the method of lines (MOL) and the finite volume method (FVM) which, eventually, yields a numerical solution. The former is an old method which was discovered by Russian mathematicians many years ago and has been expertly reviewed by Liskovets (1965), while the latter was devised by Patankar (1980). Briefly, the original MOL seeks to replace a parabolic partial differential equation in two equations of first order. Although, the discretization procedure of the second-order derivative may be achieved by standard finite-difference analogs, it is preferred to adopt the FVM developed by Patankar (1980). The implementation of the hybrid methodology requires finite volumes of finite height and infinite width (Figure 1). As a result of this combination, each equation participating in the system of first order, ordinary differential equations governs the behavior of the dependent variable θ_P representing a finite volume. Consequently, omitting the peripheral details, equation (6) may be easily converted into

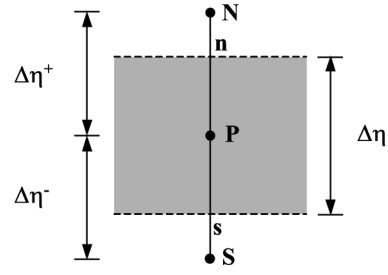

Figure 1.
Finite volume adopted for implementing the MOL

a system of first order, ordinary differential equations. An equation for a typical line P has the form:

$$\frac{\mathrm{d}\theta_{\mathrm{p}}}{\mathrm{d}z} = a_N\theta_N + a_P\theta_P + a_S\theta_S \tag{14}$$

where the coefficients are expressed by the relations:

$$a_N = \frac{8u_{\mathrm{m}}^{+}\left(\frac{1}{Pr} + \frac{\varepsilon_{\mathrm{M}}}{\nu}Pr_{\mathrm{t}}\right)_N\eta_N}{Reu_P^{+}\Delta\eta^{-}(\eta_N^2 - \eta_S^2)} \tag{15a}$$

$$a_S = \frac{8u_{m}^{+}\left(\frac{1}{Pr} + \frac{\varepsilon_{\mathrm{M}}}{\nu}Pr_{\mathrm{t}}\right)_S\eta_S}{Reu_P^{+}\Delta\eta^{-}(\eta_N^2 - \eta_S^2)} \tag{15b}$$

$$a_P = -(a_N + a_S) \tag{15c}$$

Furthermore, the prevailing initial conditions are:

$$\theta = 1, \quad \text{at all lines } P \tag{16}$$

The system of equations (14)-(16) is solved by the Runge-Kutta integration scheme. With the objective of reducing lengthy numerical computations for a turbulently moving fluid of this nature, the grid points are positioned nonuniformly in the radial direction, compressing the grid points near the tube wall. The choice of nonuniform intervals in the radial direction permits the reduction of the number of ODE's saving some time in the numerical calculations. This approach could be accomplished by implementing the radial coordinate transformation suggested by Roberts (1971):

$$\eta = \frac{(\beta + 2\alpha)\prod^{(\bar{\eta}-\alpha)/(1-\alpha)} - \beta + 2\alpha}{(2\alpha + 1)[1 + \prod^{(\bar{\eta}-\alpha)/(1-\alpha)}]} \tag{17a}$$

where:

$$\prod = \frac{\beta + 1}{\beta - 1} \tag{17b}$$

This transformation depends on two parameters: a refinement parameter, α and a streching parameter, β. By assigning a value of $\alpha = 0$, the mesh will be refined near the wall only. In fact, η and $\bar{\eta}$ are the dimensionless radial coordinates in the computational and physical plane, respectively, as shown in Figure 2. For turbulent flows, the numerical value of the parameter β is chosen such that at least one line lies inside the viscous sublayer in the thermal entry region.

Discussion of the numerical results

For the hydrodynamic part, the numerically determined friction factors compared satisfactorily with those obtained from Petukov's formula (White, 1991) applied to a circular tube with an smooth surface:

$$f = (0.79\ln Re - 1.64)^{-2}, \quad 10^4 < Re < 5\times 10^6 \tag{18}$$

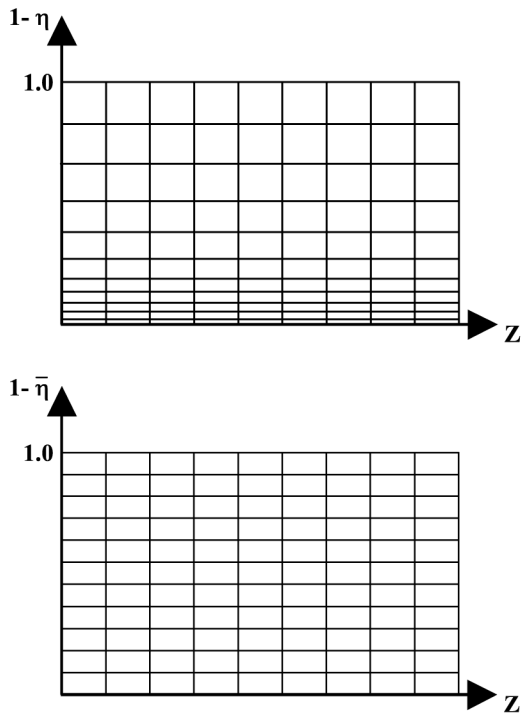

Figure 2.
Radial coordinate transformation

Because of the friction factor is a global quantity, this comparison provides ample confidence to the computed turbulent velocity profiles.

The computed asymptotic Nusselt number, Nu_∞, furnishes the data for the Nu_∞-surface in Figure 3 which serves to illustrate the influence of Re and Pr separately.

Also, the Nu_∞ values showed good agreement with the predictions using the Notter and Sleicher (1972) correlation equation:

$$Nu_\infty = 4.8 + 0.016 Re^{0.85} Pr^{0.93} \tag{19}$$

which has a range of validity: $0.004 < Pr < 0.1$ and $Re < 5 \times 10^5$.

For the remaining part of the discussion of results, thermal quantities will be presented for a metallic liquid with a $Pr = 0.03$ (mercury). Plots of the asymptotic Nusselt number in terms of Re are shown in Figure 4. Here, the characteristic straight-line behavior in log-log coordinates is manifested. The variations of Nu_∞ vs Re using the models of Azer and Chao (1960) and Dwyer and Tu (1969) for the turbulent Prandtl number are plotted using a solid and a dashed line, respectively. An envelope pattern is observed surrounding the experimental data of Sleicher and Rouse (1975), which is shown for comparison purposes.

Unquestionably, the quantity of most practical interest in thermally developing metallic flows through isothermal tubes is the mean bulk temperature and its relationship to the heat transferred between the bulk of the flow and the tube walls.

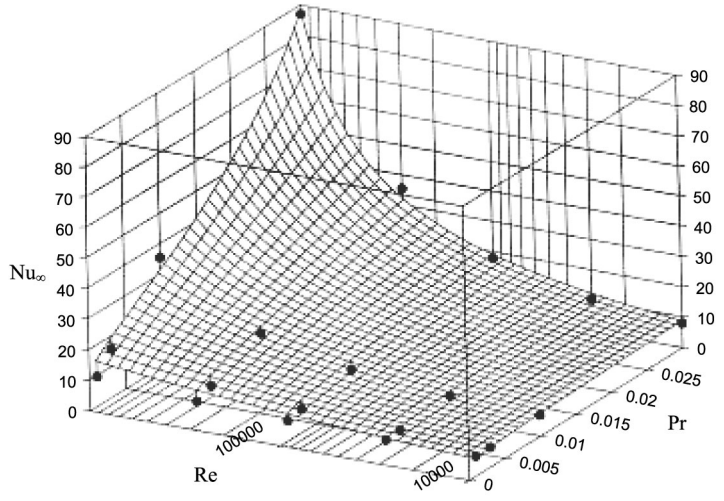

Figure 3.
Surface for the asymptotic Nusselt number varying with the Reynolds and the Prandtl numbers

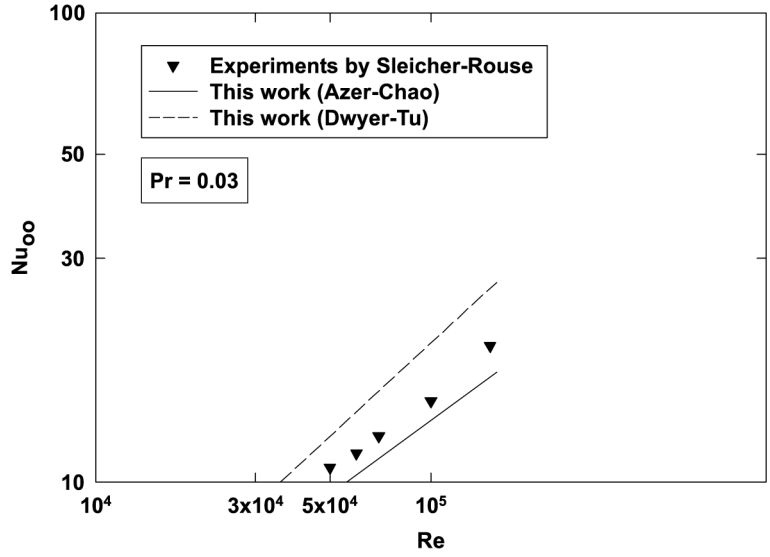

Figure 4.
Asymptotic Nusselt numbers: comparison between predictions (lines) and experiments (symbols) for $Pr = 0.03$

In this regard, results for the distributions of mean bulk temperature and local Nusselt number for a metallic liquid are shown in Figures 5 and 6. In these figures, there are two sets of curves: one gradually sloping downward which corresponds to θ_b and is referred to the right ordinate. The other, sharply sloping downward, corresponds to the ratio Nu/Nu_∞ and is referred to the left ordinate. Both curves share the same abcissa. Figure 5 shows the variation of the mean bulk temperature and Nusselt number ratio for $Re = 10^5$ using the models of Azer and Chao (1960) and Dwyer and Tu (1969) for the turbulent Prandtl number. For purposes of validation, these predictions are

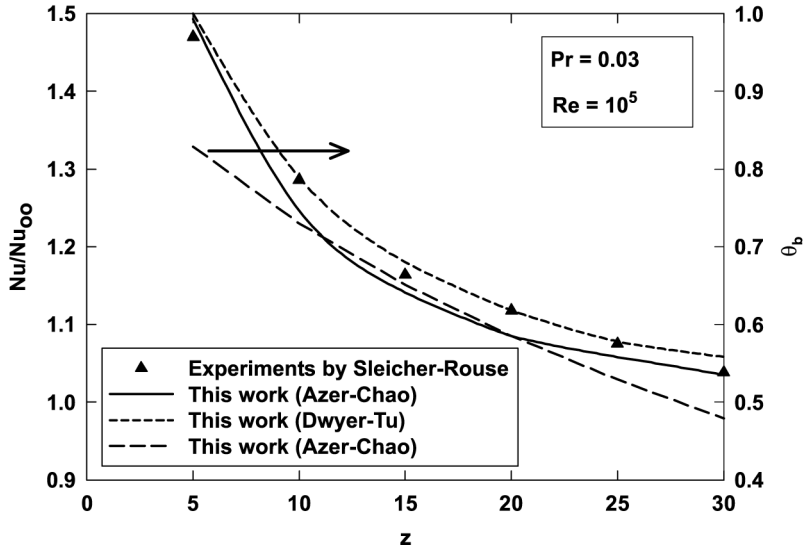

Figure 5.
Nusselt number distributions: comparison between predictions (lines) and experiments (symbols) for $Pr = 0.03$, $Re = 10^5$. Also shown is the mean bulk temperature (use right y-axis)

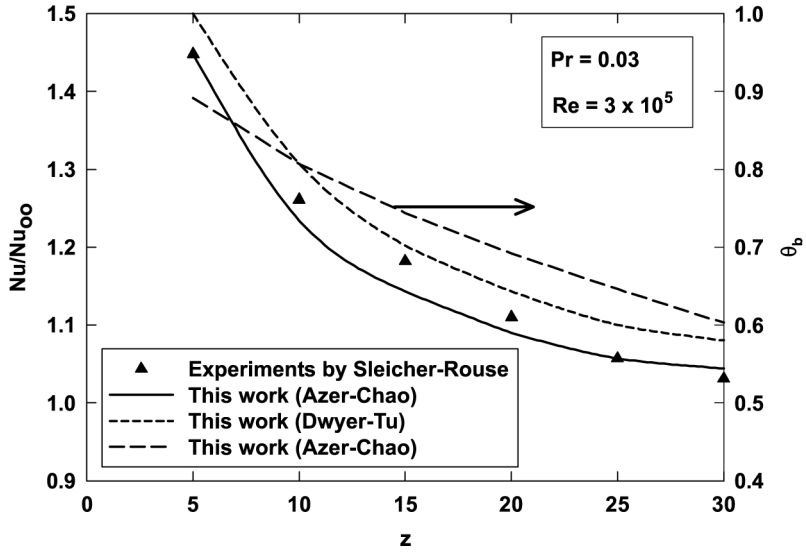

Figure 6.
Nussel number distributions: comparison between predictions (lines) and experiments (symbols) for $Pr = 0.03$, $Re = 3 \times 10^5$. Also shown is the mean bulk temperature (use right y-axis)

compared with the experimental data obtained by Awad (1965) also. The mean bulk temperature shows its characteristic monotonic decreasing behavior and is insensitive to the model adopted. In contrast, the trend observed for Nu/Nu_∞ is quite interesting: the dashed curve, based on the model of Dwyer and Tu (1969) establishes an upper bound whereas the solid curve, based on the model of Azer and Chao (1960) specifies a lower bound. In general, both sets of curves show a good quantitative behavior. The Nusselt number, Nu, being a local thermal quantity is dependent on the model adopted.

Numerically determined results and experimental data for a higher Reynolds number, $Re = 3 \times 10^5$, are shown in Figure 6. The curves exhibit the same basic features as those of Figure 5 which were discussed before. Again, both curves provide an envelope for the experimental data points.

On the whole, the foregoing comparisons indicate a remarkable level of agreement using a rather coarse grid consisting of ten unequally spaced lines for calculations of turbulent convective flows. No appreciable differences in the global quantities were detected when the number of lines was increased to 20. For the same combination of *Re* and *Pr*, the average CPU time is approximately doubled for this latter run.

Concluding remarks

An extremely simple computational procedure has been developed for analyzing entry region heat transfer to turbulent metallic liquids flowing inside tubes. The computational procedure employed the FVMOL which reformulates the partial differential energy equation with variable coefficients into a system of first order energy equations with constant coefficients. Essential quantities such as the turbulent velocity profile, the mean bulk temperature distribution and the local Nusselt number distribution are readily obtained with a Runge-Kutta integration algorithm. Upon reviewing the various comparisons with the experimental measurements and with highly accurate finite-difference solutions, there appears to be ample evidence that the Runge-Kutta based numerical results provided by the present hybrid procedure are themselves of high accuracy with a minimum computational effort.

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